Rational Exponents and Roots

In this lesson, you will . . .

* Review the laws of exponents.
* Learn that radicals and exponents are inverse operations.
* Understand how the meaning of rational exponents follows from extending the properties of exponents from integer exponents to rational exponents.
* Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Discuss with a partner: What is the *meaning* of the expression $2^{3}$? What is the *value* of the expression $2^{3}$? How do you know?

Recall that in an exponential expression of the form $b^{n}$, $b$ is called the base and $n$ is called the exponent. You can say that $b$ is raised to the $n$th power. For example, $5^{4}$ would be read as “five raised to the fourth power.” But what does this *mean*?

In expanded form, we would write $5^{4}=5∙5∙5∙5$. So the exponent is short-hand notation for repeated multiplication. Raising five to the fourth power means multiplying 5 times itself four times.

So what about $5^{\frac{1}{2}}$? How can we multiply 5 by itself one-half times?

In today’s investigation, we will find the meaning behind this expression.

But first let’s review what else you know about exponents. In previous courses, you learned properties for rewriting expressions involving exponents. Let’s look at some patterns and see if we can rediscover those properties.

**Multiplying exponential expressions with like bases:**

Work with a partner to fill in the blanks, and then study the pattern to determine the property, or shortcut, for multiplying exponential expressions with like bases.

$2^{3}∙2^{4}=\left(2∙2∙2\right)\left(2∙2∙2∙2\right)=2∙2∙2∙2∙2∙2∙2=2^{7}$

$$x^{5}∙x^{12}=\left(x∙x∙x∙x∙x\right)\left( \right)=$$

$10^{2}∙10^{5}=\left( \right)\left( \right) $=

 $x^{m}∙x^{n}=$

When multiplying exponential expressions with like bases, you should \_\_\_\_\_\_\_\_\_\_\_\_\_ the exponents.

**Dividing exponential expressions with like bases**

Work with a partner to fill in the blanks, and then study the pattern to determine the property, or shortcut, for dividing exponential expressions with like bases.

 $\frac{4^{5}}{4^{2}}=\frac{4∙4∙4∙4∙4}{4∙4}=\frac{4}{4}∙\frac{4}{4}∙4∙4∙4=1∙1∙4∙4∙4=4∙4∙4=4^{3}$

 $\frac{x^{8}}{x^{6}}=\frac{ }{ }=$

$$\frac{(0.94)^{15}}{(0.94)^{5}}=\frac{ }{ }=$$

$$\frac{x^{m}}{x^{n}}=$$

When dividing exponential expressions with like bases, you should \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ the exponents.

**Raising a power to a power**

Work with a partner to fill in the blanks, and then study the pattern to determine the property, or shortcut, for raising an exponential expression to a power.

$(2^{3})^{4}=(2^{3})(2^{3})(2^{3})(2^{3})=2^{3+3+3+3}=2^{12}$

$$(x^{5})^{2}=\left(x^{5}\right)\left( \right)=x^{\\_\\_\\_\\_\\_\\_+\\_\\_\\_\\_\\_\\_}=x^{}$$

$(10^{2})^{6}=\left( \right)\left( \right)\left( \right)\left( \right)\left( \right)\left( \right) $=

 $(x^{m})^{n}=$

When raising an exponential expression to a power, you should \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ the exponents.

**Negative exponents**

Work with a partner to fill in the blanks, and then study the pattern to determine the meaning of a negative exponent.

 $\frac{2^{3}}{2^{4}}=\frac{2∙2∙2}{2∙2∙2∙2}=\frac{2}{2}∙\frac{2}{2}∙\frac{2}{2}∙\frac{1}{2}=1∙1∙1∙\frac{1}{2}=\frac{1}{2^{1}}$

 $\frac{4^{5}}{4^{7}}=\frac{4∙4∙4∙4∙4}{4∙4∙4∙4∙4∙4∙4}=\frac{}{}∙\frac{}{}∙\frac{}{}∙\frac{}{}∙\frac{}{}∙\frac{}{}∙\frac{}{}=\frac{}{^{}}$

$$\frac{x^{3}}{x^{8}}=\frac{ }{ }=\frac{}{}∙\frac{}{}∙\frac{}{}∙\frac{}{}∙\frac{}{}∙\frac{}{}∙\frac{}{}∙\frac{}{}=\frac{}{^{}}$$

If we apply the previous rule about dividing exponential expressions with like bases, each of the above problems would look like:

$$2^{3-4}=2^{-1}$$

$$4^{5-7}=4^{}$$

$$x^{3- 8 }=x^{}$$

Find the following values for powers of 3 and record in the table below. Use a calculator if needed. Convert any decimal values to fraction form.

|  |  |
| --- | --- |
| Power of 3 | Value |
| $$3^{4}$$ |  |
| $$3^{3}$$ |  |
| $$3^{2}$$ |  |
| $$3^{1}$$ |  |
| $$3^{0}$$ |  |
| $$3^{-1}$$ |  |
| $$3^{-2}$$ |  |

Study the table and the previous examples to determine the meaning of a negative exponent. What is the meaning of a negative exponent?

$$x^{-m}=$$

**Back to rational exponents . . .**

In order to understand the meaning of rational exponents, consider the following table of exponential expressions and their values:

|  |  |
| --- | --- |
| Exponential Expression | Value |
| $$2^{8}$$ | 256 |
| $$2^{4}$$ | 16 |
| $$2^{2}$$ | 4 |
| $$2^{1}$$ | 2 |
| $$2^{\frac{1}{2}}$$ |  |

What is the pattern for the exponents? $8, 4, 2, 1, \frac{1}{2}, …$

What is the pattern of the simplified values? $256, 16, 4, 2, …$

If you continue the pattern, what would be the value of $2^{\frac{1}{2}}$?

What is the meaning of $x^{\frac{1}{n}}?$

$$x^{\frac{1}{n}}=$$

EXAMPLE

Rewrite the following exponential expressions as radical expressions.

$$5^{\frac{1}{3}}$$$$y^{\frac{1}{2}}$$

$$16^{\frac{1}{4}}$$

Rewrite the following radical expressions as exponential expressions.

$$\sqrt[3]{x}$$

$$\sqrt{6}$$

$$\sqrt[5]{32}$$